$$\frac{\partial f(a)}{\partial x} = -2 \cdot \sum_{i=1}^{n} (x_i - a) = 0$$

$$\Rightarrow -2 \cdot \sum x_i + 2 \cdot n \cdot a = 0$$

$$\Rightarrow a = \frac{\sum x_i}{n} = \bar{x}_n \longrightarrow$$ noktası min. yada Max. dur

ikinci türev

$$\left.\frac{\partial^2 f(a)}{\partial x^2}\right|_{a=\bar{x}_n} = 2n > 0$$ old. için $a = \bar{x}_n$ noktası minimuma sahiptir.

- **Teorem:** $X_1, \ldots, X_n$, μ ve σ^2'li bir kitleden örneklem olsun.

a.) $E(\bar{X}_n) = \mu$, b.) $V(\bar{X}_n) = \frac{\sigma^2}{n}$, c.) $E(S_n^2) = \sigma^2$

İspat: a.) Beklenen değer operatörünün lineer olmasından dolayı,

$$E(\bar{X}_n) = E\left(\frac{1}{n}\sum x_i\right) = \frac{1}{n}\sum E(x_i) = \frac{1}{n}\sum \mu = \frac{n\mu}{n} = \mu$$

b.) Benzer olarak,

$$V(\bar{X}_n) = V\left(\frac{1}{n}\sum x_i\right) = \frac{1}{n^2}\cdot\sum V(x_i) = \frac{n\cdot\sigma^2}{n^2} = \frac{\sigma^2}{n} //$$

c.) $$E(S_n^2) = E\left[\frac{1}{n-1}\cdot\sum (x_i - \bar{x}_n)^2\right] = \frac{1}{n-1}E\left[\sum (x_i^2 - 2\cdot\bar{x}_n\sum x_i + \bar{x}\right.$$

$$-2n\bar{x}_n^2 + n\bar{x}_n^2$$

$$= \frac{1}{n-1} E\left[\sum x_i^2 - n\bar{x}_n^2\right] = \frac{1}{n-1}\cdot\left[\sum E(x_i^2) - n\cdot E(\bar{x}_n^2)\right]$$

$$= \frac{1}{n-1}\cdot\left[n\cdot(\sigma^2+\mu^2) - n\cdot\left(\mu^2 + \frac{\sigma^2}{n}\right)\right]$$

$$= \frac{1}{n-1}\cdot\left[n\sigma^2 + \cancel{n\mu^2} - \cancel{n\mu^2} - \sigma^2\right]$$

$$= \frac{(n-1)\cdot\sigma^2}{n-1} = \sigma^2 //$$

$V(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \sigma^2 + \mu^2$

$[E(\bar{x})]^2 = \mu^2 + \frac{\sigma^2}{n}$

$[E(\bar{x})]^2 = \cancel{V(\bar{x})}\, E(\bar{x}^2) + V(\bar{x})$